Session 13

Quantum physics

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Contents

Welcome

Welcome to session 13 of the Physics programme. In this section we will look at the ideas present in quantum physics.

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Learning Objectives

- Explain the notion of wave-particle duality, including the wave nature of the electron
- Discuss the interpretation of the wave function and recognise the inadequacy of the Copenhagen interpretation
- Show knowledge of the Schrodinger equation and its role in microscopic physics
- Apply the Schrodinger equation to describe a ‘particle in a box’ and to explain qualitatively the spectrum of the harmonic oscillator
- Describe what is meant by tunnelling and its application in a STM
- Use the Uncertainty Principle to estimate quantum effects, including line width
- Show an awareness of macroscopic quantum effects

The Problem

This session is unusual In that our problem will occupy only first three sections because we shall devote section 4 to a discussion of the meaning of quantum theory - a subject of current research.

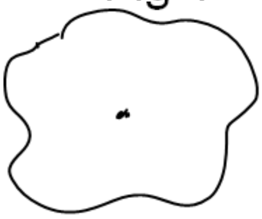
The problem is:

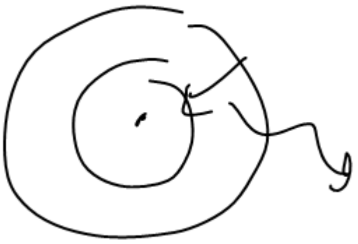
If atoms are mainly empty space, why are solids hard? Put another way, why can't we squeeze the space out of an atom. Clearly the problem has to do with understanding the atomic structure of matter. What we need is a more complete theory to underpin the Bohr model. What evidence do we have for the basis of such a theory?

Matter Waves

The first clue was found by de Broglie. His argument was as follows: if light is both waves and particles, perhaps objects such as electrons are both particles and waves! In particular can the electron inside the atom behave as a wave? Direct evidence that electrons have wavelike properties came only in 1926 with the experiments of Davisson and Germer on electron diffraction, but assuming such wavelike properties De Broglie drew the picture in figure 1 of a wave just fitting inside its orbit as a possible way of explaining the Bohr model.

Fig 1



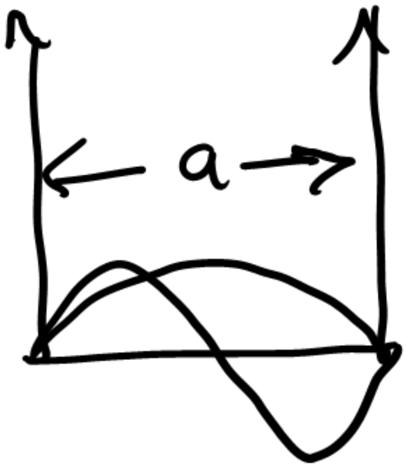


The way in which this can be developed into a theory of the hydrogen atom is mathematically complicated, so we'll consider a simplified model atom in which the electron is confined to a certain region of space just as in a real atom, but with a different confining potential.

Particle in a box (model atom)

Our simple atom is called a particle in a box. How does it compare to the real thing? First, it's one dimensional – the electron can move only along the x-axis instead of in three dimensions. This means we have to ignore any angular momentum the real electron might have. Second, the force that keeps the electron in the atom is not provided by the charge on a nucleus, but by the walls of the box. The electron bounces between the walls. Finally, and the last serious simplification: it's not really an electron, which has a property called spin, but some structureless particle with the mass of the electron.

Between the walls of the box the electron moves freely. At the wall it is subject to an instantaneous reversal of its velocity. This is convenient because it means that the speed is constant – it can be only plus or minus some quantity v. But an instantaneous reversal of velocity can happen only if the momentum changes instantaneously so there is an infinite rate of change of momentum and hence an infinite force. This can come only from a potential that changes at the wall from 0 to infinity. So the electron in this model atom is situated inside an infinite potential well.



$$\psi(x) = \sin\left(\frac{n\pi x}{a}\right)$$

$$n = 0,\ 1,\ 2\ \ldots$$

Following de Broglie's idea we ask what waves fit into this well. If one end of the box is at $x = 0$ and the other at $x = a$ then the spatial variation of the waves is of the form $\sin(n\pi\, x/a)$ with the possible values for n being 1,2,3… and so on. This function vanishes at both $x=0$ and $x=a$ so the waves do fit. You should check that you see why. We've called this spatial function ψ which is the conventional symbol for a sub atomic wave.

What are the energies of the electron in this atom? One might guess that $E = h\upsilon$ or $h\lambda/c$ where λ is the wavelength in be Broglie's picture. This doesn't work. The reason is that $E=h\upsilon$ applies to photons that move at the speed of light, but not to relatively slow moving electrons. Instead we keep $p = h/\lambda$ but put:

$$p = \frac{h}{\lambda} = \hbar k, \quad E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m} \qquad \text{(non – relativistic)}$$

The wavelengths of the sine wave is $2n\pi/\lambda$ or nk where k is the wavenumber equal to $2\pi/\lambda$l. We end up the with energy in equation (1):

$$E = \frac{h^2\pi^2 n^2}{2ma^2} \qquad (1)$$

For the ground state, n = 1, this is exactly the same as the Bohr model. For higher states it differs from the Bohr model quite strikingly: For the particle in a box the energies are proportional to n^2 not $1/n^2$ as in the Bohr atom.

What information do we have about the particle in this picture? All the information must be contained in the wave function ψ, but rather than doing the maths, we'll extract as much as we can from the physics.

What do we know about the momentum of the particle? The standing wave in the box does not have a direction of travel associated with it – so the magnitude of the particle's momentum is $\hbar k$, but it can be either $+\hbar k$ or $-\hbar k$; thus the momentum is uncertain to $2\hbar k$.

$$\Delta p = 2\hbar k$$

Where is the particle? The wave function simple tells us that the particle is in the box where the wave is non-zero. So there is an uncertainty of a in the position x.

$$\Delta x = a$$

Multiplying the uncertainties:

$$\Delta x \Delta p = 2\hbar \frac{2\pi}{a} a = 2h$$

Our conclusion is that if a particle in a box is described in terms of a wave $\psi(x)$ then it is impossible to obtain complete knowledge of its position and momentum. Heisenberg generalised this to all circumstances. Heisenberg's uncertainty principle is the key to quantum theory and states that complementary variables such as x and p cannot both have definite values; they must satisfy $\Delta x\Delta p \sim h$. So this says that the more accurately we know, say the position, the less accurately do we know its momentum and vice versa.

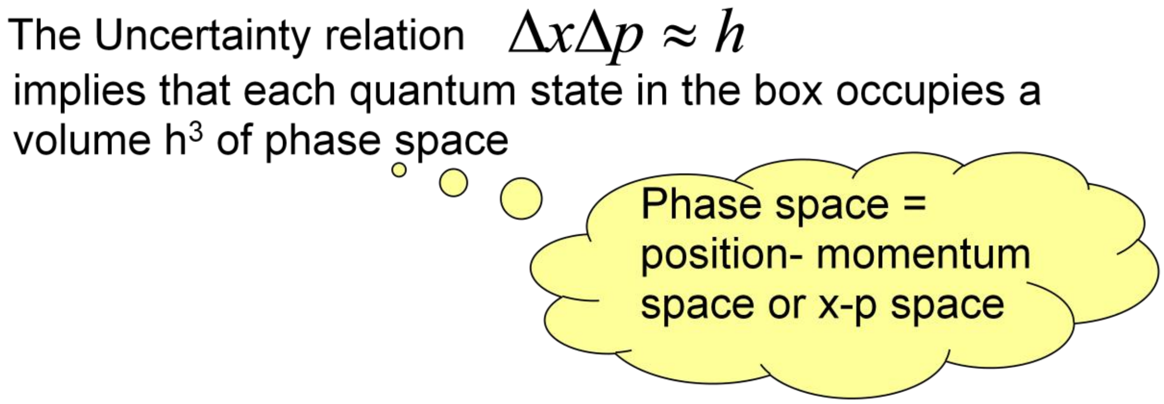
Heisenberg's Uncertainty Principle: $\Delta x \Delta p \geq h$

Comparison with the Bohr Model

Let's compare the particle in a box with the Bohr model. We know that $\Delta x\Delta p \sim h$ and also that $\Delta x \sim a$. We also know that $\Delta p \sim p$. which is the square root of 2mE. This information can be rearranged to give $a\sqrt{2mE} \approx h$. Thus we get $E = \frac{h^2}{2ma^2}$ which is just the energy of the ground state in the Bohr model. In this picture the electron cannot be at rest at r=0 in the ground state because that would violate the uncertainty principle.

Check this

Of course, the higher energy states do not correspond to the Bohr picture at all – here the details depend on the shape of the confining potential. This is a general feature of quantum mechanics – order of magnitude results depend only on the general property of the Uncertainty Principle, but for any more detail one needs to do specific calculations, as we'll see.



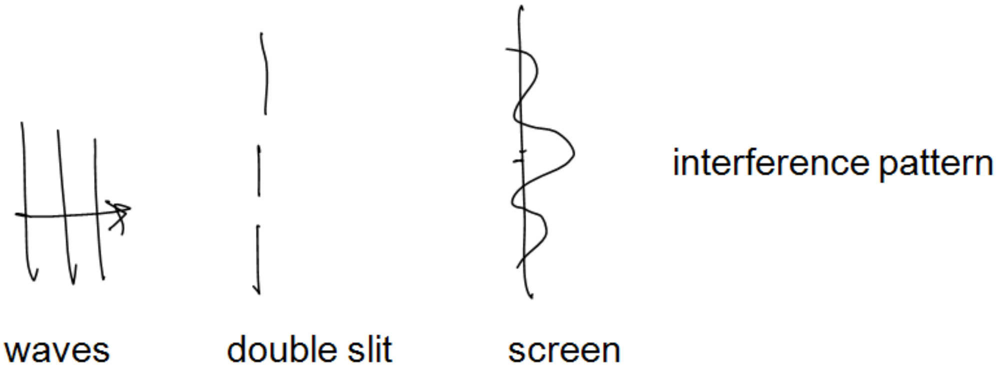
The Uncertainty relation $\Delta x\Delta p \approx h$ implies that each quantum state in the box occupies a volume h^3 of phase space

Phase space = position- momentum space or x-p space

We can also calculate the area of x-p space occupied by each state. To go from one state to the next the wavelength changes by π/a, hence the momentum changes by $h\pi/a$. Thus the state occupies a range $h\pi/a$ in momentum. It also occupies a range a in space. So the area in x-p space is $h\pi$ or of order h. This is a general result; in two dimensions the volume per state would be h^2 and in three dimensions h^3.

Example: Young's Double Slit Experiment

Let's look at another example of the Uncertainty principle in action. In a Young's two slit experiment a collimated beam of light is passed through a screen with two slits and the resulting interference pattern observed on a screen.



If we do not know which slit a photon passes through the uncertainty in position is the slit separation, *a*. The uncertainty in momentum is related to the width of a interference peak; we know these occur at $\sin\theta \sim \theta = \lambda/a$; so $\Delta p = p\theta = h/\lambda \times \lambda/a$; Thus $\Delta p \Delta x = h$.

Suppose we try to determine the position of a photon more accurately than this by measuring which slit it goes through. The simplest way of doing this is to close one slit, at which point the interference pattern disappears. But it can also be done without perturbing the photon. This is nowadays a practical experiment. Even so, as soon as we do this, the interference pattern once again disappears.

Summary

- Both light and matter are postulated to behave as particles and waves, depending on the observations being made.
- Treating the electron as a wave gives rise to a model atom as an electron in an infinite potential well.
- The energy levels in this model are quantised and the ground state has non-zero energy.
- The Uncertainty Principle sates that complementary variables such as position and momentum cannot both have precise values in a quantum state
- There is one quantum state per h^3 of position-momentum (phase) space

SAQs

1. What is the wavelength of a 100MeV electron?

   (a) 4000 nm (b) 2 x 10^{-6} nm (c) 10^{-5} nm

2. For a particle in a box in the nth excited state the momentum is between +hnπ/a and -hnπ/a so the uncertainty in momentum is n times larger than in the ground state. Which of the following is true?

   (a) the uncertainty in position remains the size of the box and the U P does not apply

   (b) the uncertainty in position remains the size of the box: the UP gives the minimum uncertainty and is still valid

   (c) the uncertainty in position is decreased because the particle is less likely to be found near the walls

The answers appear on the following page

Answers

1. (a) Incorrect – you have probably lost a factor of c

   (b) Incorrect – you have probably introduced a factor of 2π

   (c) Correct $p = h/\lambda = \sqrt{2mE}$; with $m = 0.5$ MeV/c^2 and $h = 4.1 \times 10^{-15}$ eV s

2. (a) Incorrect. The UP always applies to quantum systems

   (b) Correct. The UP says that the product of uncertainties $\Delta p \Delta x$ can be no less than h: it can be greater

   (c) Incorrect: all we know about the particle is that it is confined to the box.

The Bohr Atom

We stated earlier that the general results of quantum mechanics could be obtained from order of magnitude estimates using the Uncertainty principle. We'll see several more examples of this, starting with the Bohr model.

Bohr Atom from the Uncertainty Principle

$$E = \frac{p^2}{2m} - \frac{e^2}{4\pi\varepsilon_0 r}$$

$$p = \frac{h}{r}$$

$$E = \frac{h^2}{2mr^2} - \frac{e^2}{4\pi\varepsilon_0 r}$$

For a minimum energy: $$0 = \frac{dE}{dr} = -\frac{2h^2}{2mr^2} + \frac{e^2}{4\pi\varepsilon_0 r^2}$$

$$r = \frac{h^2}{m}\frac{4\pi\varepsilon_0}{e^2}$$

This shows how the uncertainty principle determines the size of the atom

Complete the calculation by determining the ground state energy *E* and compare this with the Bohr result

Particle in a box: Spectra

Let's see how much more we can find out from our particle in a box model atom. We know the particle is described by a wave:

$$\psi(x) = \sin\left(\frac{n\pi x}{a}\right)$$

This is usually referred to as the wave function of the particle. What can we say about the energy of the particle? – We've see that this is definitely:

$$E = \frac{n^2\pi^2\hbar^2}{2ma}$$

We're thinking of this particle as an electron in an atom, and here we can see that the only way this enters the discussion is through the mass m. That's because we've replaced the Coulomb field between the proton and electron by the box.

To express the fact that the wave function corresponds to a definite energy we say that the wave function, ψ, is an eigenstate of energy – a somewhat unfortunate mixture of German and English which is the now established terminology. The corresponding energy itself is said to be an eigenvalue to emphasise the fact that there is no uncertainty associated with it.

Looking at the dimensions of Planck's constant, energy x time, we see that energy and time are complementary variables. The uncertainty in energy being zero, the Uncertainty principle therefore tells us that the uncertainty in time must be infinite – that is the wavefunction contains no information about time.

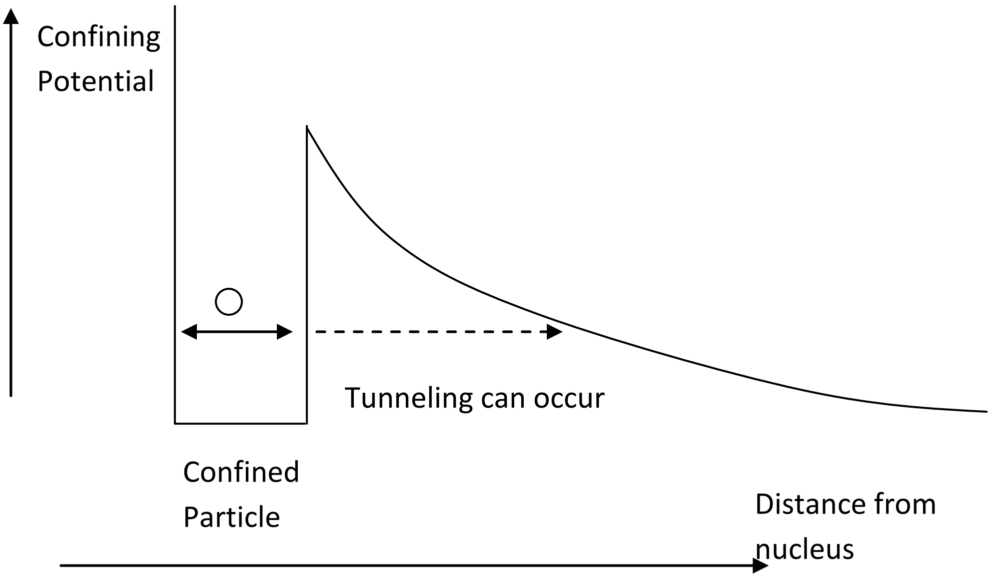
$$\Delta E\Delta t \sim h \text{ or } \Delta\upsilon\Delta t \sim 1$$

This is indeed the case – the wavefunction describes a steady state independent of time. This is important: the electron wave in this case is not a sort of smeared out particle moving around in the box. It IS a standing wave.

Now, what happens when the electron makes a transition from one state to another by absorbing or emitting a spectral line? The decay of an excited state is governed by the radioactive decay law: $N = N_0\ e^{-t/\tau}$ Here τ is the mean lifetime of the excited state: the decay typically takes place within a time interval τ. Typically, the lifetime of an excited state might be around 10^{-8} seconds. If we have this time information, it must therefore be impossible for us to have no uncertainty in the energies involved – in this case the energy of the emitted photon. We expect $\Delta E \sim h/\Delta t$ or h/τ. We thus predict that spectral lines with have an irreducible minimum spread in energy – that is they will appear to have a width in a spectrograph. This is indeed borne out by experiment, although there are other factors which also contribute to the observed width.

Notice how we have again been able to make an order of magnitude estimate using just the Uncertainty principle.

Particle in a box: Tunnelling



The infinite potential jump and infinite force exerted by the walls on the particle is an unrealistic feature of the model. Let's see what happens if we reduce the potential to a finite discontinuity.

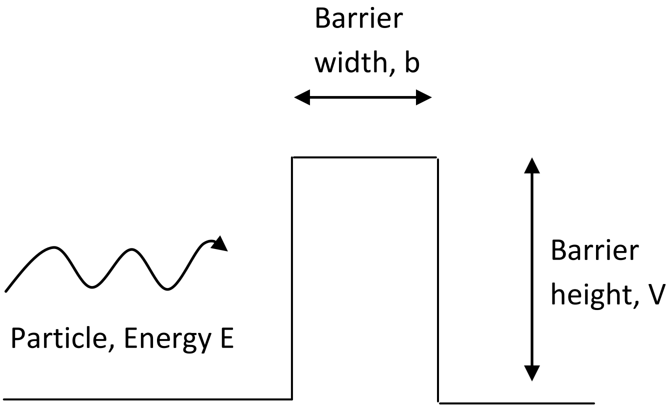
Classically the discontinuity in the potential represents an infinite force – since the force is the gradient of the potential which is zero in the box and infinite at the walls. So a classical particle still bounces around in the box. For a quantum mechanical particle on the other hand there is an uncertainty associated with the energy that allows it to be larger than potential barrier for a short enough time. The particle therefore has a probability of escaping from the box, or, in other words of being detected beyond the walls of the box. This is for example how one can describe alpha decay – the escape of an alpha particle from the confines of the nucleus. The process of passing through a classically impenetrable barrier is called quantum mechanical tunnelling.

As always we can estimate the scale of the effect using the uncertainty principle. The distance beyond the box in which we are relatively likely to find the particle is given by $\Delta x \sim h/\Delta p \sim h/\sqrt{2mE}$ since all we know about p is that it equals + or - $\sqrt{2mE}$. Finally we can estimate the order of magnitude of the energy as V. This gives us the tunnelling distance.

The tunnelling of a charged particle out of a potential well will give rise to an electrical current which can be detected. The amount of current will depend on both the shape of the potential and how close the pick-up is to the edge of the potential well. This is how a scanning tunnelling microscope works – it can be used to generate pictures of single atoms, although what it actually measures is the potential (or force field) surrounding the atom.

Uncertainty Principle estimate of the tunnelling probability

In this section we have another example of tunnelling where we can use the uncertainty principle to estimate the probability of tunnelling. In this case we have a particle coming in from the left impacting on a potential barrier.



$$\Delta E = (V - E)$$

and $$\Delta t = \frac{b}{v}$$

where $$v = \frac{p}{m} = \frac{\sqrt{2m(V-E)}}{m}$$

The height of the barrier represents its potential energy. Classically if the kinetic energy of the incoming particle is less than the barrier energy the particle is reflected. This is because the kinetic energy inside the barrier is E-V which would be negative, hence impossible to achieve classically. Quantum mechanically the uncertainty in energy allows the particle to skip over the barrier.

We can estimate the tunnelling probability as:

$$\exp(-\Delta t \Delta E / h) = \exp\left\{ -\frac{(V-E)^{1/2} m^{1/2}}{bh} \right\}$$

The probability falls off exponentially the greater the departure required from the minimum uncertainty. Now, a reasonable estimate of ΔE is (V-E). This gives the momentum and hence the speed as in equation (1). The time of flight across the barrier is therefore t = b/v for a barrier of depth b. This is our estimate for Δt since this is the time the particle spends in the classically unattainable state. The final result is the main factor that emerges from a proper calculation.

Summary

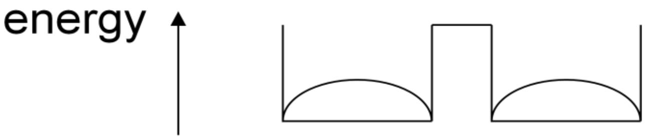
- Order of magnitude estimates can be obtained in quantum theory from the Uncertainty Principle

  Examples include:

    - The size of an atom and the ground state energy as in the Bohr model
    - The probability of tunnelling through a potential barrier
- A particle in a box can serve as a simple model of an atom

SAQs

1. The diagram shows two potential wells; initially one particle is put into well 1 and another into well 2. For the sake of argument assume that the particles can be distinguished by their spin, say. Which of the following is a (or are) true.



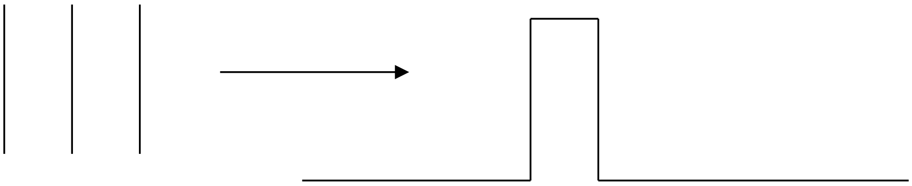
(a) particle 1 cannot tunnel to well 2 because it is already occupied

(b) tunnelling from well 1 is unaffected by the presence of a particle in well 2

(c) tunnelling cannot occur by symmetry because tunnelling both ways makes no difference

(d) the system will end up with a equal probability of each particle in each well (1 in 1, 1 in 2 , 2 in 1, 2 in 2)

2. A steady stream of particles of definite momentum is incident on a barrier which classically they cannot cross. In quantum mechanics which of the following is true?



(a) all particles will be reflected because quantum theory and classical physics have to be consistent

(b) there is a probability of tunnelling so sometimes there will be a particle to the right and sometimes not

(c) because we're dealing with a steady incident stream there is a constant probability of finding a particle to the right of the barrier but no information on where it is.

3. A typical excited level in hydrogen has a spontaneous Einstein coefficient for decay of A ~ 10^8 per second. The width of the typical emission line is:

(a) 0

(b) 10^8 Hz

(c) 4.1 x 10^{-7} eV

(d) 10^{-8} s

The answers appear on the following page

Answers

1. (a) and (d) are correct: the particles are independent and the final state has to be symmetrical because there is nothing in the set-up that associates a particular particle with a particular well.

2. (a) Incorrect: certain process that are forbidden classically are allowed in quantum mechanics

   (b) Incorrect: The set-up describes a steady stream so the outcome of measurement cannot depend on time.

   (c) Correct: the particles have definite momentum hence entirely undetermined position. There is a constant probability of finding a particle to the right of the barrier.

3. (a) The width of the line cannot be zero because of the uncertainty principle – a purely harmonic wave of a single definite frequency cannot have a beginning and an end.

   (b) Correct $\Delta \upsilon = 1/\Delta t = A$

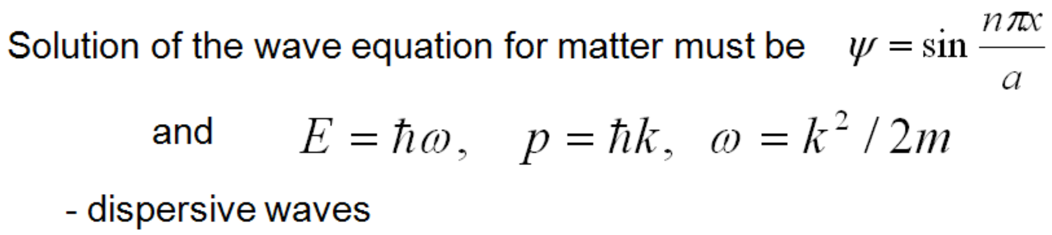
   (c) Correct $\Delta E = h/\Delta t$ is a valid form of the Uncertainty principle

   (d) Incorrect: $1/A$ is the lifetime of the state not the width

Schrodinger Equation

For a complete theory we need an equation that can describe matter waves, just as we had equations in session 3 to describe water waves and electromagnetic waves. One way to guess this equation is to look at the dispersion of these waves; that is the relation between frequency and wavelength. We have E = hυ or hω/2π and p = h/λ or hk/2π. Therefore E = $p^2/2m$ becomes $\omega = k^2/2m$.

That is:



Solution of the wave equation for matter must be $\psi = \sin\frac{n\pi x}{a}$

and $E = \hbar\omega, \quad p = \hbar k, \quad \omega = k^2/2m$

- dispersive waves

The waves are therefore dispersive – different frequencies travel at different speeds and so an initially sharp pulse will spread out. The Schrodinger equation is the simplest equation with this property. For a free particle it takes the form (1):

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \qquad \text{(1) (The Schrodinger Equation for a free particle)}$$

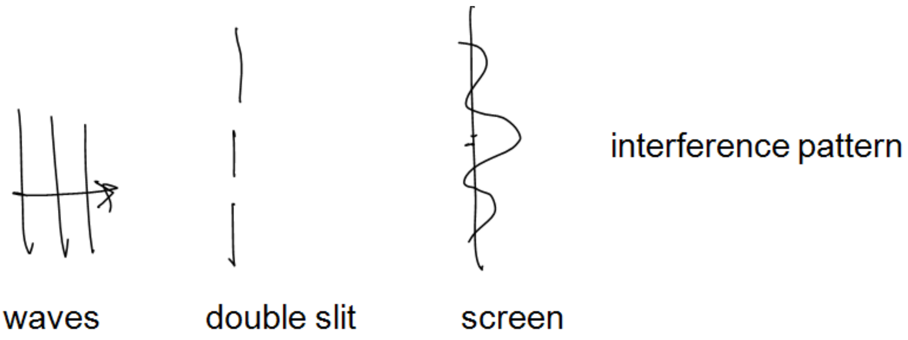
Notice that in contrast to the waves we were dealing with in session 3 it has only a first derivative with respect to time and also that it contains i, the square root of -1 . An alternative form of the equation applicable to stationary states (states that are unevolving in time) is given by (2) and is sufficient for students who are not familiar with complex numbers.

$$E\psi = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} \qquad \text{(2) (Time independent form)}$$

For a particle in a potential V(x), that is one subject to a force, -dV/dx, E is replaced by (E-V) because in this case $p^2/2m$ = (E-V) instead of E.

What does ψ measure?

So we now have an equation for a wave function $\psi(x)$, but we haven't said what ψ measures. Let's return to the double slit experiment only this time imagine that it is single photons at a time that are sent through the apparatus.



This experiment can be carried out; what is found is that it is impossible to predict where on the screen a single photon will end up, although as more and more photons hit the screen their points of impact form the original diffraction pattern. Thus the wave ψ that describes the photon behaviour must be related to the probability of finding a photon in a certain region of space. In fact, $l\psi l^2$ dx gives the probability of finding the photon in the region dx.

In fact $|\psi|^2$ dx is the probability of finding the particle in dx

Superposition

One of the key aspects of the Schrodinger equation is that it is linear – each term involves just ψ or its derivatives. This means that given two solutions of the Schrodinger equation, that is two possible configurations of matter waves, we can get a third by adding them together with arbitrary weightings as we've done in equation (1). We call this process the superposition of ψ_1 and ψ_2. What does this represent?

$\psi(x) = a\psi_1(x) + b\psi_2(x)$, where:

$$\psi_1 = \sin\left(\frac{n\pi x}{a}\right)$$

$$\psi_2 = \sin\left(\frac{m\pi x}{a}\right)$$

Let's assume that ψ_1 and ψ_2 are eigenfunctions, let's say of energy. When we measure the energy associated with ψ we don't get something in between the values for ψ_1 and ψ_2. In fact, since ψ is not an eigenfunction of energy we can't get a definite values of energy at all. What we get is sometimes the energy of ψ_1 and sometimes that of ψ_2. In fact, the probability of measuring ψ_1 is a^2 and of ψ_2 it is b^2.

We can turn back again to spectra to get a nice application of this. The electron jump between levels ψ_1 and ψ_2 that gives rise to a spectral line can be describe by a wave function ψ as in (1), except that a and b themselves are time dependent (equation (2)). We start in state 1 (say) so initially a=1 and b=0, and we end in state 2, so finally a=0 and b=1. But we can't measure anything other than 1 or 2 – we can't see the transition in between! -- We never see the electron jump.

Schrodinger equation for a particle in a potential

$$-\frac{h^2}{2m}\frac{d^2\psi}{dx^2}+V(x)\psi = E\psi$$

If we want to describe the matter wave of a particle subject to forces we have seen that we have to add a potential energy to the Schrodinger equation. One common example is the quantum harmonic oscillator. Thinking back to session 3 you may recall that the potential energy of a harmonic oscillator is $\frac{1}{2} m\omega^2x^2$. We're not going to attempt to solve the Scrodinger equation in this case in order to find the stationary states because it can't be done by any elementary methods. Instead we'll content ourselves with the answer: it is that the oscillator energies are quantised and that the energy eigenvalues for the harmonic oscillator are $(n + \frac{1}{2})\, h\omega/2\pi$ where n = 0, 1, 2… Often you will see this written as $(n + \frac{1}{2})\, \hbar\omega$ where $\hbar = h/2\pi$.

$$E = (n+\tfrac{1}{2})\hbar\omega \qquad n = 0,1,2,3,... \qquad (3)$$

Note that the lowest energy is not 0 but $1/2\ \hbar\ \omega$. The $1/2\ \hbar\ \omega$ is called the zero point energy. Quantum mechanically the oscillator cannot come to rest. Why do you think this is?

Why do we need the $\frac{1}{2}\hbar\omega$? Hint: Uncertainty principle

As an aside note that the electromagnetic field also has a zero point energy – even in the vacuum there are energy fluctuations. These are what keep the electron in the lowers Bohr orbit from falling into the nucleus. They are also the source of spontaneous emission: in fact spontaneous emission is emission stimulated by the zero point energy of the electromagnetic field.

Why is matter solid?

We can now give an answer our initial question: why is matter solid that is incompressible, given that it consists mainly of empty space?

Imagine that we can increase the pressure on some material as much as we like. As the pressure is increased the atoms will lose their electrons and become ionised. We can work out the pressure required for this – as always with order of magnitude estimates involving pressure it's best to think of this as an energy per unit volume rather than the equivalent force per unit area. To strip atoms of their electrons the pressure has to supply the ionisation energy per unit volume.

Next we estimate the compressibility of matter. As usual, we use the uncertainty principle to do this. Let there be n electrons per unit volume in a volume V. Assuming that we can neglect the electrostatic interaction of the electrons, their energy is just the total kinetic energy.

$$K = V\frac{dP}{dV}$$

Pressure ~ Energy per unit volume: $= n\frac{p^2}{2m}$

This is therefore nV x $p^2/2m$ where p is the momentum. We use the uncertainty principle to estimate the momentum per particle. Each electron is known to be confined to the volume V or R^3, so $V^{1/3}$ is the uncertainty in its position; its momentum is therefore $h/V^{1/3}$ or h x $n^{1/3}$. The energy is then given by equation (3). The pressure is the energy per unit volume to within a numerical factor so is proportional to $n^{5/3}$.

Uncertainty Principle $h \approx V^{1/3} p \approx n^{-1/3} p$

So Pressure = $\frac{h^2}{2m} n^{5/3}$ This gives $K = 10^{11}$ Pa

Putting in the numerical values gives a bulk modulus of around 10^{11} Pascals. This means that a pressure of 10^9 Pa is required to produce a 1% change in volume.

This order of magnitude estimate somewhat disguises the fact that the high incompressibility of matter is due to the Pauli Exclusion Principle. Because of this we have to treat the electrons as independent particles which each satisfy the uncertainty principle.

Without the exclusion principle the electrons would, in effect, join into a single so called condensate, which would satisfy the uncertainty principle as a single entity with a much lower energy density.

Summary

- The Schrodinger equation governs the (non-relativistic) behaviour of matter at the quantum level
- The solution of the Schrodinger equation for a particle is the wave function ψ that contains all the information available about the particle
- $|\psi|^2$ dx is the probability of finding the particle in the region dx.
- If ψ_1 and ψ_2 are possible wave functions then so is $a\psi_1 + b\ \psi_2$
- Matter is solid because of the pressure generated by electrons as a result of the exclusion principle. This can be estimated from the Uncertainty Principle

SAQs

1. True or False:

   (i) Pressure alone cannot ionise an atom; a high temperature is always required

   (ii) Matter is solid because the nuclear force is repulsive

   (iii) Matter is solid because the electrons fill the spaces between ions

   (iv) Matter is solid because it takes a lot of energy to squeeze out electrons

2. Which of the following are possible free particle wave functions

   (a) $A\sin kx + B\cos kx$ (b) e^{kx} (c) $\sin kx$ (d) $\cos kx$

   $k = \sqrt{2mE}/\hbar$

The answers appear on the following page

Answers

1. (i) false

   (ii) false

   (iii) false

   (iv) false

2. See the text for the correct answers.

Interpretation of Quantum Theory

Having solved our problem using quantum theory in the next few sections we shall try to understand what the theory means. This is an open question. However, we shall certainly be able to say what the theory doesn't mean.

A modern view that is certainly consistent with quantum theory but which many people find deeply unattractive, is the many-worlds interpretation. In this picture the many possibilities described by the wave function actually exist (the many worlds), but we explore only one path of these possibilities. An alternative modern approach attempts to take quantum theory seriously as a complete description but to include not only the system under study, but its inevitable interaction with the rest of the world as well. This enables the quantum/classical boundary to be a matter of calculation, but still doesn't provide a complete picture.

So the final message is that quantum theory certainly works – much of modern technology depends on this – but we don't fully understand how.

Heisenberg microscope

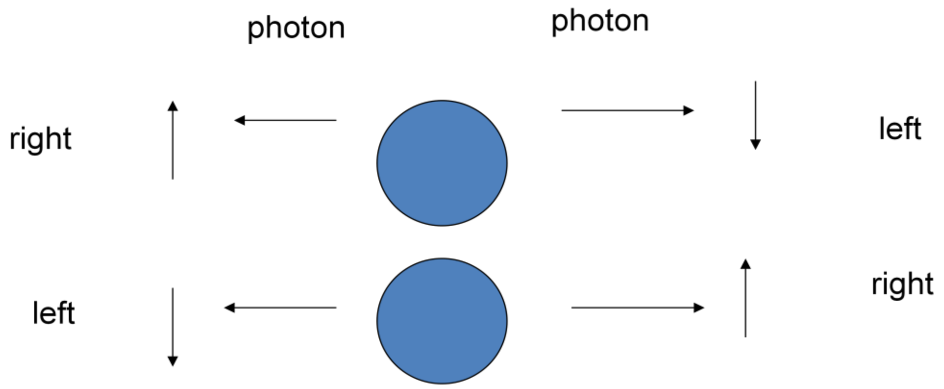
One way in which it is often presented is in terms illustrated by the Heisenberg gamma ray microscope. This story goes like this. If we want to measure a position accurately we have to measure small distances. Therefore we need high momenta. If we're using electromagnetic radiation this means we use gamma rays. But if we bombard a particle with gamma rays to try to find out where it is, we shall certainly disturb its momentum. Similarly to tie down its position to within a distance λ requires waves of wavelength $< \lambda$ - the collision perturbs its momentum by h/λ so we no longer know the momentum.

Hence it is impossible to determine an accurate position and momentum simultaneously. This is simply not a valid argument. The gamma microscope might not be effective, but laser cavities certainly could be: the point however is that it doesn't matter how little we disturb the system – we still cannot measure a position and momentum more accurately than the uncertainty principle allows. This is encapsulated in the formalism of quantum theory: the wavefunction cannot describe a particle with both a position and a momentum. A plane wave has a definite momentum, but fills space. If we try to confine the wave function it can no longer be a pure harmonic wave and hence cannot have a definite momentum.

However, this is not the general issue – we can measure without perturbation.

The lack of information does not arise from uncontrollable disturbances introduced by measurement; this is not the problem in general! Even if we do not disturb the state we cannot measure more than the information contained in the wavefunction, which is never complete information on position and momentum.

The EPR experiment – two photon decay of an excited atom



Imagine that an atom decays emitting two photons back to back. If one is circularly polarised in one direction (clockwise say) the other will be polarised in the opposite sense (anticlockwise). This is because angular momentum must be conserved in the decay. Before any measurement of its polarisation the photon on the right is equally likely to be left or right circularly polarised. The wavefunction is therefore an equal mixture of both states. But a measurement of the state of the photon on the right immediately gives us certain information on the state of the photon on the left, without perturbing the left state in any way. Similar arguments can be constructed for momentum and position.

This is known as the Einstein-Rosen-Podolsky paradox. The wave function on the left changes instantaneously from a mixture of polarisation sates to a definite polarisation once the measurement is made on the right. This successfully undermines the original Copenhagen interpretation because it means that the wavefunction cannot represent a real physical wave. At best the interpretation must be changed so that the wavefunction represents only our knowledge of the system.

Copenhagen Interpretation

The Copenhagen interpretation is more subtle but wrong. In this picture there is a separation between the quantum world described by the wavefunction ψ and the classical world described by measurements on the system in the state ψ. This is usually described in terms of a collapse of the wavefunction on measurement. Because, although ψ describes probabilities initially, once a measurement is made we know which of the alternatives in ψ is actually realised. So ψ no longer describes the system. Apparently a measurement changes the wavefunction, although there is nothing in the theory that describes the mechanism of this change. Schrodinger's cat is a thought experiment highlighting the absurdity of this Copenhagen interpretation. The cat is inside a box with a radiaoactive atom and geiger counter connected to a phial of poison. When the radioactive nucleus decays the geiger counter detects the decay and releases the poison. After one half life the nucleus may or may not have decayed: quantum mechanically the system is a superposition of a parent and daughter nucleus. (Compare our discussion of the decay of an excited atom in quantum mechanics.) But it is therefore also a superposition of a live and a dead cat. While one might accept the superposition of the two nuclei – and the success of quantum theory at this level leaves us no choice but to do so -- it is quite another thing to envisage the superposition of a live and dead cat. Thus even the non-realist interpretation of the wave function as merely a bearer of information fails. As ever, the fundamental problem is the lack of a description of the interface between the quantum and classical pictures.

Modern Views

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Decoherence: interaction with the environment brings about collapse and defines the classical-quantum interface.

Summary

- Quantum theory is extraordinarily successful at describing the microworld
- Despite this it is incomplete: in particular the classical – quantum boundary is not specified by the theory
- This has led to a large number of attempts to interpret the relation of the theory to the world
- The Copenhagen interpretation of wave function collapse as a result of a classical measurement is untenable as illustrated by the EPR
- Experiment and Schrodinger's cat
- Contemporary interpretations include the many worlds picture and the decoherence approach

SAQs

1. A wavefunction ψ is known to correspond to a state having a definite energy E. Which of the following is true:

   (a) Measurement of the energy will always yield E

   (b) The state also has a definite momentum

   (c) The state has a definite position

2. A wavefunction $\psi = a\phi_1 + b\phi_2$ with φ_1 and φ_2 corresponding to states of definite energy E_1 and E_2. Which of the following is true?

   (a) Measurement of ψ can never yield a value for energy

   (b) energy E_1 is measured $\frac{a^2}{a^2+b^2}$ of the time

   (c) Measurement of ψ always yields the energy $(E_1 + E_2)/2$

   (d) Measurement of ψ yields E_1 a/(a+b) of the time

3. Suppose a system of two particles with eigenfunctions φ_1, φ_2 and ψ_1, ψ_2, is in the state $\psi = a\psi_1\phi_1 + b\psi_2\phi_2$. What is the probability of measuring ψ_1?
   (a) $a^2/(a^2 + b^2)$ (b) $a^2\varphi_1^2 / (a^2\varphi_1^2 + b^2\varphi_2^2)$ (c) $a/(a+b)$

   If the result of a measurement is ψ_1 what is the probability of finding the system in the state φ_1?
   (a) $a^2/(a^2 + b^2)$ (b) 1 (c) $a/(a+b)$

The answers appear on the following page

Answers

1. (a) is correct. The state may also have a definite momentum but this is not necessarily the case. (For a plane wave it will be true, for an atomic energy level it is not).

2. (a) Incorrect: a measurement of the energy will always yield a value – although this may differ form one identical measurement to another
   (b) correct: the squares of the coefficients are the probabilities that the system will be an the corresponding eigenstate.
   (c) incorrect: measurements can only yield eigenvalues, here E_1 and E_2
   (d) Incorrect : the squares of the coefficients are probabilities

3. (a) correct: the squares of the coefficients give the relative probabilities
   (b) Incorrect: the combined states are φ_1 ψ_1 and φ_2 ψ_2 with coefficients *a* and *b.*
   (c) Incorrect: the squares of the coefficients give the relative probabilities

Additional Problems

Please read these as further ideas, rather than problems to be solved. How does what you've learned so far apply?

Problem 1: Quantum Computing

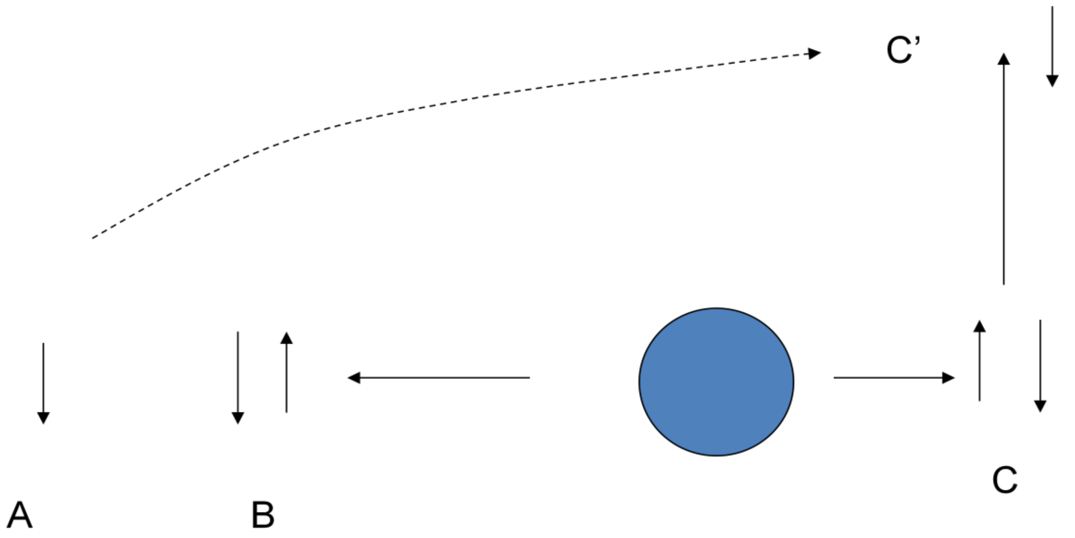
In this section we'll touch ever so briefly on the notion of quantum computing. The basic idea is that any operation on the wave function ψ is an operation on each of the many possibilities it describes when we write it as a sum of eigenfunctions, ψ_1, ψ_2 etc.

$$\psi = a_1\phi_1 + a_2\phi_2 + ...$$

The operations on the eigenfunctions are simultaneous so we have a massively parallel computer. We read out the result of computation by the EPR method of measuring an entangled state in order to obtain the result without perturbing it. There is some way to go to constructing a practical computer on these lines but minor computations appear to demonstrate the principles involved.

Problem 2: Teleportation

In the figure the state of A is transferred to C via the entangled state B using the EPR effect. B is first entangled with C; it is then used to measure A which automatically determines C′ as in the EPR experiment. A is thereby teleported to C′.



Teleportation is impossible classically. Quantum mechanically it is also impossible to copy a system. Teleportation works by using an entangled state to read out the original state. The original state is destroyed in the process, so it doesn't violate the no-copy rule. Teleportation does not violate any known laws: in particular information isn't sent faster than light. It's just a convenient way – in principle – of moving information around with less emphasis on transport of physical objects.

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